

# Division algebras and algebraic groups: structure and cohomology

Andrei Rapinchuk

**Abstract.** The talk is a tribute to the Saint Petersburg algebraic school. We will begin with Faddeev's theorem on the Brauer group of the field of rational functions, and discuss the extension of its most essential part to arbitrary reductive algebraic groups proposed by Raghunathan and Ramanathan. We will sketch a recently found short proof of the Raghunathan-Ramanathan theorem that uses building-theoretic techniques. These techniques are also instrumental in the analysis of the structure of groups of points of algebraic groups - the area to which Nikolay Vavilov has made major contributions. We will first describe finite subgroups of the groups of points of reductive algebraic groups over polynomial rings of characteristic zero, and then turn to the problem of bounded generation of Chevalley groups over rings of arithmetic type by root subgroups. Here we will emphasize the case where the base ring is the coordinate ring of a geometrically integral smooth curve over a finite field, and survey the results of Kunyavskii-Plotkin-Vavilov. In the concluding part of the talk, we will introduce the genus of a simple algebraic group, and highlight one application of the Raghunathan-Ramanathan theorem to the genus problem. We will close with the notion of motivic genus proposed by Merkurjev and the relevant results of Izboldin, Karpenko and Vishik.

Andrei Rapinchuk  
Department of Mathematics, University of Virginia,  
P.O. Box 400137,  
Charlottesville, VA 22904 , USA  
e-mail: [asr3x@virginia.edu](mailto:asr3x@virginia.edu)