

# Visualizing finite groups and algebras

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Dessins d'enfants were introduced by Alexander Grothendieck in a rather non-traditional way, see [Gro1984]. The main objects of this theory are the graphs embedded in compact oriented surfaces in such a way that the complements are homeomorphic to disjoint unions of open disks. When the category of dessins d'enfants is neatly defined, it turns out to be equivalent to the arithmetic-geometric category of the so-called *Belyi pairs* and to certain categories of finite groups, see, e.g., [ShVo1990] or [Shab2016].

After the brief general introduction some examples of the relations between dessins d'enfants and certain algebraic structures will be considered.

Following [AKSS1995] and [Mati2016], the visual images of the Mathieu groups  $M_{11}$  and  $M_{23}$  will be presented.

For a prime-power  $q$  the visualization of the finite field  $\mathbb{F}_q$  by means of the complete graph with  $q$  vertexes will be discussed. More precisely, it will be explained that *every choice of a generator of the multiplicative group  $\mathbb{F}_q^\times$  defines a highly symmetric dessin*; the field can be restored by this dessin [Shab2025].

The case of  $\mathbb{F}_7$  will turn out to be closely related to the *octonions*.

## References

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