Linear Algebraic Groups vs Automorphism Groups

Ivan Arzhantsev

In this talk we discuss the similarities and differences between linear algebraic groups and automorphism groups of affine algebraic varieties. We are especially interested in the group of special automorphisms $SAut(X)$, i.e., the normal subgroup of $Aut(X)$ generated by all one- parameter additive subgroups. We consider the Tits Alternative and the flexibility property for affine varieties. The group $SAut(X)$ acts in the regular locus of a flexible variety X infinitely transitively, that is, any finite collection of smooth points can be sent to any finite collection of smooth points of the same cardinality. Using flexibility, we show that every non-degenerate toric variety, every homogeneous space of a semisimple group, and every variety covered by affine spaces admits a surjective morphism from an affine space. Applying the ellipticity property introduced by Mikhail Gromov in 1989, we prove that a complete algebraic variety X is an image of an affine space if and only if X is unirational.

This is a joint result with Shulim Kaliman and Mikhail Zaidenberg.

Ivan Arzhantsev
Faculty of Computer Science,
HSE University,
Moscow, Russia
e-mail: arjantsev@hse.ru

The research is supported by the RSF-DST grant 22-41-02019.