

Gröbner-Shirshov bases, Dehn’s algorithm and small cancellation rings

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In Group Theory there exists a class of problems on constructing groups with exotic properties, e.g., infinite Burnside groups, Tarskii Monster etc.. There exists a universal approach for these problems which is called “Iterated small cancellation theory”. Its essence is that the desired group is presented as a direct limit of hyperbolic groups (or even stronger small cancellation groups). In Ring Theory there exists a similar class of problems on constructing algebras with exotic properties. Particularly, we are interested in the following old problem: does there exist a division algebra infinite dimensional over its center with finitely generated multiplicative group (posed by Kaplansky, Lvov and Latyshev in 1970s). For algebras so far there is no uniform approach to construct such “monster” objects. In papers [1, 2, 3] we made a significant progress towards such an approach. Namely, we introduced new techniques which allow to work with ring relations using the idea of small cancellation from Group Theory, gave a definition of small cancellation rings, and described their properties and further applications.

References

- [1] A. Atkarskaya, *Small cancellation rings are non-amenable*, preprint (2023), 15 pages, available at <http://arxiv.org/abs/2210.13303>.
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- [3] A. Atkarskaya, A. Kanel-Belov, E. Plotkin, E. Rips, *Construction of a quotient ring of \mathbb{Z}_2F in which a binomial $1 + w$ is invertible using small cancellation methods*, Contemporary Mathematics, IMCP, Volume 726, 2019, pages 1–76, available at <https://arxiv.org/abs/1807.10070>.

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