Matrix evaluations of non-commutative polynomials

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The talk is dedicated to the memory of Nikolai Vavilov. We extend his ideas on group word mappings to the ring case. It is joint work with Sergey Malev and Louis Rowen.

The general statement is as follows. Let $P(x1, \dots, x_n)$ be a non-commutative polynomial in matrices of order n. What can be said about the set of its values?

I. Kaplansky and I. V. Lvov posed the question (see Dniester Notebook) that the set of values of a multilinear polynomial is a vector space (in this case it coincides either with zero, or with the space of all matrices, or with the space of traceless matrices, or with scalar matrices). The solution of Kaplansky's problem even for second-order matrices over a square-closed field turned out to be quite non-trivial and deep. It got many references. Recently it was solved for Calley algebra.

Questions related to equations in matrices, in addition to their applied significance, are related to the construction of an algebraically closed field, to the freedom theorem: if we add a new non-commutative variable and a relation where it is involved, this will not lead to the appearance of new relations. There are a number of deep problems related to the set of values of words in a group, in particular in second-order matrices.

We discourse state of art of this problem, some partial results, Lie algebra case and some relations with group theory.

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