

Algebras of permutations

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The group algebras $C[S_m]$ of the symmetric groups and, especially, their centers $ZC[S_m]$ play a crucial role in the study of the structure of these groups and their representations. Moreover, they are widely used in the analysis of geometry of moduli spaces of algebraic curves. In particular, Hurwitz numbers,

which enumerate ramified coverings of the projective line with prescribed ramification types, can be interpreted as connection coefficients of certain classes generating $ZC[S_m]$ (A. Hurwitz, 1891). This understanding led to a relationship between Hurwitz numbers and integrable systems of mathematical physics (A. Okounkov, 2000). Recently, in the study of real ramified coverings of the projective line, different versions of these algebras, which are called algebras of transition types, naturally appeared (M. Kazarian, S. Lando, S. Natanzon, 2021). Other recent appearances of algebras associated to permutations are related to weight systems, which are a combinatorial counterpart of finite type invariants of knots. Originally, weight systems were defined as functions on chord diagrams, which can be understood as permutations of special kind, namely, involutions without fixed points (V. Vassiliev, 1990). Last years, an idea due to M. Kazarian allowed for extending them to arbitrary permutations and to developing efficient algorithms for computing weight systems (M. Kazarian, S. Lando, Z. Yang). The talk will be devoted to describing these objects and to discussing various possible relationships between them.

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