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Automorphisms of twisted Chevalley groups over rings

Elena I. Bunina

Abstract. Let R be a commutative ring with identity, equipped with an automorphism θ of order dividing 2 or 3. We assume that $2 \in R^\times$ and, in the triality case, also that $3 \in R^\times$. I will present a local-global classification of automorphisms of elementary and full twisted Chevalley groups of types ${}^2A_\ell$, $\ell \geq 3$, ${}^2D_\ell$, $\ell \geq 4$, 2E_6 , and 3D_4 , for every invariant isogeny type.

The proof combines the normal structure of twisted Chevalley groups with local rigidity and affine gluing. An automorphism first acts on the maximal congruence quotients. After a global normalization of the standard relative-root frame, its action on the relative root subgroups is recovered from an automorphism of the coefficient ring. Consequently, outside a small list of exceptional cases, all automorphisms are compositions of the familiar inner, diagonal, ring, graph, and central factors.

Over arbitrary rings two additional phenomena occur. In the low-rank unitary types 2A_3 and 2A_4 , there is an additive family of exceptional automorphisms supported on the fixed residual characteristic-three locus. In absolute type D_4 , local graph data must be assembled into compatible global graph-ring automorphisms. In triality, the coefficient factor is governed by preservation of the triality trace; the global theorem uses a separate residual and relative local triality input.

As a concrete consequence, we obtain a complete description of the automorphisms of the hyperbolic special unitary group $SU_4(R, \theta; Q_4)$ over every commutative ring with $2 \in R^\times$.

The results are joint work with Deep H. Makadiya.

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Reductive Group Schemes over LG-Rings and Applications

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Abstract. This talk is joint work with Erhard Neher and is based on the paper published in the Nikolai Vavilov Memorial Issue of the European Journal of Mathematics [G-N]. We study reductive group schemes over local-global (LG) rings and develop a local-global approach to their structure. It applies to conjugacy results for parabolic subgroups and for maximal split subtori.

Introduction and Motivation

The foundational treatise SGA3 by Demazure-Grothendieck establishes the theory of reductive group schemes over a base ring R . Several central results—existence of maximal tori, conjugacy of parabolic subgroups, and transitivity properties—are, however, only proved in the special case where R is a *semilocal* ring. Our purpose is to extend most of these results to the strictly larger class of *LG-rings* (local-global rings).

LG-rings. The concept of an LG-ring, introduced by McDonald and Waterhouse [M-W] and first studied in depth by Estes and Guralnick [E-G], axiomatizes a key property of semilocal rings: a polynomial in several variables represents a unit of R if and only if it represents a unit over every residue field $R/\mathfrak{m}$ (equivalently, over every localization $R_{\mathfrak{m}}$). This “local-global” principle for units is the origin of the name. The class of LG-rings properly contains:

- all semilocal rings (hence all fields and all artinian rings);
- rings that are von Neumann regular modulo their Jacobson radical;
- zero-dimensional rings;
- the ring of all algebraic integers (and all real algebraic integers);
- any finite R -algebra or integral extension of an LG-ring.

Notable non-examples include $\mathbb{Z}$ and polynomial rings $R[X]$ over an integral domain.

Motivation. The subject of LG-rings fell into near-oblivion until it was revived by Garibaldi, Petersson, and Racine [G-P-R1, G-P-R2] in their work on quadratic forms and Jordan algebras. The present talk provides a geometric approach to LG-rings and systematically extends the SGA3 toolkit, opening the way to applications in the theory of forms and Azumaya algebras.

1. Geometric Characterization of LG-Rings

The key technical innovation is a geometric reformulation of the LG-property that enables the transfer of SGA3 arguments from the semilocal to the LG setting.

Geometric characterization. *Let R be an LG-ring, M a finite locally free R -module, and $U \subset \mathbf{W}(M)$ an open quasi-compact subscheme of the affine scheme $\mathbf{W}(M) = \mathrm{Spec}(\mathrm{Sym}(M^*))$ associated with M . Then:*

$$U(R) \neq \emptyset \iff U(R/\mathfrak{m}) \neq \emptyset \text{ for every maximal ideal } \mathfrak{m} \triangleleft R.$$

This local-to-global lifting of rational points is the engine of the paper. The proof leverages the fact that for a quasi-compact open U defined by a finitely generated ideal I , one can construct an auxiliary linear polynomial that represents a unit modulo every $\mathfrak{m}$; the LG-property then promotes it to a unit over R . A companion result handles the *primitive condition*—rings that are LG with all residue fields infinite—where R -density of U suffices.

A first application is a streamlined proof that finite projective R -modules of constant rank over an LG-ring are free, recovering $\mathrm{Pic}(R) = 0$ by an inductive argument using unimodular elements.

2. Existence of Maximal Tori

It generalizes Grothendieck’s theorem on the existence of maximal tori in reductive group schemes [SGA3, XIV, 3.20] from semilocal rings to LG-rings. The proof follows the SGA3 approach via regular elements of the Lie algebra $\mathfrak{g} = \mathrm{Lie}(G)(R)$. An important step is

Let G be a reductive group scheme over an LG-ring R satisfying the primitive condition, or let G be adjoint. Then $\mathfrak{g}$ admits a regular element.

The main result is the following:

Let G be a reductive group scheme over an LG-ring R . Then G admits a maximal R -torus.

A further issue is that every parabolic subgroup admits a maximal torus.

3. Parabolic Subgroups: Conjugacy and the Tits Index

We come now to the conjugacy theorems for parabolic subgroups.

Let G be a reductive group scheme over an LG-ring R . Two parabolic subgroups of G of the same type are conjugate under $G(R)$.

This generalizes the classical semilocal result. The proof again uses the geometric characterization of LG rings: the relevant quotient scheme has $R/\mathfrak{m}$ -points for all $\mathfrak{m}$, hence an R -point.

It provides a unique smallest type $\mathbf{t}_{\min}$ in the set of types of parabolic subgroups of G over an LG-ring. When G is semisimple and R is connected, allowing to define the *Tits index* of G over R . It turns out that no new indices arise beyond those already known from the field case.

4. Minimal Parabolics and Maximal Split Tori

We focus now on minimal parabolic subgroups, their Levi subgroups, and maximal split tori in reductive group schemes over a *connected* LG-ring.

Let R be a connected LG-ring and G a reductive R -group scheme.

- (a) *$G(R)$ acts transitively on the set of minimal parabolic subgroups of G .*
- (b) *If G is semisimple, $G(R)$ acts transitively on the maximal split R -subtori of G .*

This transitivity is the analogue of the Borel–Tits theory over fields and is essential for defining structural invariants. As a consequence, one defines the *anisotropic kernel* of G as the derived group of a minimal Levi subgroup—well-defined thanks to the conjugacy result. This invariant plays a role analogous to the anisotropic kernel in the classical theory of algebraic groups over fields.

5. Application : Cancellation Theorems

It leads to cancellation theorems via a cohomological principle having the following consequences:

- **Cancellation of modules and Azumaya algebras in tensor products:** If $P \otimes Q_1 \cong P \otimes Q_2$ for suitable finite projective modules or Azumaya algebras, then $Q_1 \cong Q_2$.
- **Cancellation of hermitian forms:** analogues of Witt cancellation for hermitian forms over LG-rings, extending results from the semilocal case.
- **Witt cancellation of quadratic forms :** if $q_1 \perp q_3 \cong q_2 \perp q_3$ for quadratic forms q_1, q_2, q_3 over an LG-ring, then $q_1 \cong q_2$. This generalizes the earlier result of Dias [D] from her Ph.D. thesis, which was limited to a more restrictive class of rings.

These cancellation theorems are the algebraic-geometric counterparts of the classical cancellation results for forms over fields and semilocal rings, now valid over the broader LG setting.

Conclusion

We extended important results from fields and semilocal rings to the broader class of LG-rings. In particular one obtain cancellation theorems for projective modules and Azumaya algebras. They also establish cancellation results for hermitian and quadratic forms over LG-rings.

Overall, the work demonstrates the strength of LG-rings as a natural setting for generalizing classical algebraic structures and cancellation phenomena.

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Double cosets of stabilizers of totally isotropic subspaces in a special unitary group. Geometrical approach

Nikolai Gordeev

Let D be a division algebra with an involution and with the centre F and let V be a finite dimensional vector space with a symmetric or skew-symmetric hermitian form. Further, let h be isotropic and let P_u, P_v be the stabilizers of totally isotropic subspaces $u, v \leq V$ in the group $SU(D, h)$. Here we consider the “geometrical” description of double cosets $P_u g P_v$ where $g \in SU(D, h)$. Namely, such a coset is defined uniquely by the dimension and the Witt index of the space $u + g(v)$ (with one exception). The “adherence” of double cosets is also defined by geometrical parameters.

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Burnside-type algebras

Ilya Gorshkov

This talk is devoted to the construction of axial algebras and their connection with Burnside groups $B(n, p)$, where p is a prime. For a group $G = B(n, p) \cdot \langle t \rangle$, where t is an automorphism of order 2, an algebra $A(G, T, \eta)$ is constructed whose Miyamoto group is isomorphic to $G/Z(G)$, with G being finite if and only if the algebra itself is finite-dimensional which provides an algebraic criterion for the finiteness of Burnside groups. General properties of such algebras and their connection to the classical theory of axial algebras are also discussed.

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Automorphic forms, cohomology of arithmetic groups, and moduli spaces

Valery Gritsenko

Abstract. The talk will review results based on the substantive connection between the structure of generators of arithmetic groups and automorphic forms, which allows us to prove the nontriviality of higher cohomology groups.

Introduction

My work is devoted to the theory of automorphic forms, but in these notes I try to present it from the point of view of the theory of arithmetic groups. We are talking about non-triviality of the third cohomology groups of the paramodular (symplectic) groups of genus 2 and the fourth cohomology of some integral orthogonal groups of signature (2,4). Both questions are associated with the irrationality (non-negativity of the Kodaira dimension) of moduli spaces of complex varieties. In our topic, the question of special generators of orthogonal groups is important. For example, we discuss conditions under which Eichler transvections generate a stable integral orthogonal group of an indefinite quadratic lattice over the ring of integers. We will also formulate some questions in this direction.

1. The paramodular group

The *paramodular group* Γ_t ($t \in \mathbb{N}$) is the integral symplectic group of rank 2 of a symplectic form with elementary divisors $(1, t)$. It is the modular group of the moduli space $\mathcal{A}_t$ of polarized abelian surfaces with a polarisation of type $(1, t)$.

We start with natural questions related to the first and second cohomology groups of Γ_t . We note that $\Gamma_1 = Sp_4(\mathbb{Z})$ is perfect. In [2] we analysed the commutator of Γ_t for $t > 1$. We proved that

$$\Gamma_t^{ab} = \Gamma_t / [\Gamma_t, \Gamma_t] = (\mathbb{Z}/(t, 12)\mathbb{Z}) \times (\mathbb{Z}/(2t, 12)\mathbb{Z}).$$

The paramodular group Γ_t is not maximal discrete subgroup of $Sp_4(\mathbb{R})$. Its maximal normal extension is generated by special quasi-involutions (see [2])

$$\Gamma_t^* = \langle \Gamma_t, V_d \rangle, \quad V_d^2 \in \Gamma_t, \quad d|t, \quad (d, \frac{t}{d}) = 1,$$

and $\Gamma_t^*/\Gamma_t \cong (\mathbb{Z}/2\mathbb{Z})^{\rho(t)}$, where $\rho(t)$ is the number of prime divisors of t .

The situation with the third cohomology group of Γ_t is rather non-trivial. In [1] we proved that

$$\text{rank } H^3(\Gamma_t, \mathbb{C}) > 0 \quad \text{if } t \neq 1, \dots, 12, 14, 15, 16, 18, 20, 24, 30, 36.$$

More exactly, in [1] we constructed canonical differential forms on any smooth compactification of the moduli space of $(1, t)$ -polarised abelian surfaces using author's lifting construction of modular forms of weight 3 with respect to Γ_t by Jacobi cusp forms of weight 3 and index t . In other words, our method is transcendental not algebraic. For example, a non-trivial class in $H^3(\Gamma_{13}, \mathbb{C})$ for the first possible $t = 13$ is given by the lifting of the following theta-block

$$\varphi_{3,13}(\tau, z) = \frac{\vartheta(\tau, z)^5 \vartheta(\tau, 2z)^3 \vartheta(\tau, 3z)}{\eta(z)^3} \in J_{3,13}^{cusp},$$

where $\vartheta(\tau, z) = -\vartheta(\tau, -z)$ is the odd Jacobi theta-series of characteristic 2 and $\eta(z)$ is the Dedekind eta-function. (See [7] for the theory of theta-blocks.)

Problem. *For what natural t the group $H^3(\Gamma_t^*, \mathbb{C})$ is not trivial?*

For a prime p the group Γ_p^* is the modular group of the moduli spaces of $(1, p)$ -polarized Kummer surfaces (see [2]). We found the first primes with non-trivial $H^3(\Gamma_t^*, \mathbb{C})$ in [6]. They are $p = 167, 173, 223$, etc.

2. Orthogonal group of $O'(L)$

Here L is an integral even lattice, i.e. a free $\mathbb{Z}$ -module with a non-degenerate bilinear form $(\cdot, \cdot): L \times L \rightarrow \mathbb{Z}$ such that $(u, u) = u^2 \in 2\mathbb{Z}$ for any $u \in L$. The dual lattice L^* contains L . We denote its *discriminant group* by $D(L) = L^*/L$. It carries a finite quadratic forms with values in $\mathbb{Q}/2\mathbb{Z}$. The *stable orthogonal group* $\widetilde{O}(L)$ is defined as the kernel of the natural projection to the finite orthogonal group $O(D(L))$. We consider the spinor norm $\text{sn}_K: O(L \otimes K) \rightarrow K^\times / (K^\times)^2$ over a field K and define the *stable orthogonal group* and the *spinor kernel*

$$\widetilde{SO}^+(L) = \widetilde{SO}(L) \cap \ker \text{sn}_{\mathbb{R}}, \quad O'(L) = SO(L) \cap \ker \text{sn}_{\mathbb{Q}}.$$

We tell that an even integral lattice L satisfies the *Kneser condition* if its Witt index ≥ 2 over $\mathbb{R}$, L represents -2 and that $\text{rank}_3(L) \geq 5$ and $\text{rank}_2(L) \geq 6$.

Theorem. (See [4, §1].) *If L satisfies the Kneser condition then $O'(L) = \widetilde{SO}^+(L)$. Moreover, the abelianisation $\widetilde{O}^+(L)^{ab}$ is an abelian 2-group. Its order divides 2^N , where N is the number of different $\widetilde{O}^+(L)$ -orbits of (-2) -vectors in L . The same is true if we replace $\widetilde{O}^+(L)$ by $\widetilde{SO}^+(L)$ throughout.*

To proof the theorem we used the strong approximation for indefinite orthogonal groups and some considerations with reflections. We got a better result using special unipotent elements called Eichler transvections

$$t(e, a): v \mapsto v - (a, v)e + (e, v)a - \frac{1}{2}(a, a)(e, v)e \in \widetilde{SO}^+(L).$$

The *divisor* $\text{div}(l)$ of $l \in L$ is the positive generator of the ideal $(l, L) \subset \mathbb{Z}$. This is a divisor of $|L^*/L| = |\det(L)|$.

We define $E(L)$ to be the group generated by all transvections by unimodular isotropic vectors:

$$E(L) := \langle t(e, a) \mid e, a \in L, (e, e) = (e, a) = 0, \text{div}(e) = 1 \rangle.$$

Theorem. (See [4, Theorem 1.7].) *Let $L = U \oplus U \oplus L_0$ be an even lattice with two hyperbolic planes U , such that $\text{rank}_3(L) \geq 5$ and $\text{rank}_2(L) \geq 6$. Then*

$$\widetilde{SO}^+(L) = O'(L) = E(L),$$

$\widetilde{SO}^+(L)^{ab}$ is trivial and $\widetilde{O}^+(L)^{ab} \cong \mathbb{Z}/2\mathbb{Z}$.

From this algebraic result the following geometric consequences follow:

Theorem. (See [4, Theorem 5.5 and Theorem 6.7].) *Any smooth model of a compactification of the moduli space $\mathcal{A}_t$ of $(1, t)$ -polarized abelian surfaces is simply-connected. The same is true for the moduli space of $2d$ -polarized K3 surfaces.*

3. $H^4(\Gamma_{t,s}, \mathbb{C})$ where $\Gamma_{t,s} \subset O(2, 4)$

The projectivised paramodular group $\Gamma_t/\{\pm E_4\}$ is isomorphic to the stable special orthogonal group $\widetilde{SO}^+(L_t)$ of the integral lattice $\Lambda_{2t} = U \oplus U \oplus \langle -2t \rangle$ of signature $(2, 3)$ (see [2]). Constructing canonical differential forms (or nontrivial cohomology classes) for the stable orthogonal group of $\widetilde{O}^+(L)$ is much more difficult. The case of signature $(2, 19)$, corresponding to polarized K3 surfaces, was studied in [3]. In the theory of moduli spaces of polarized hyperkähler manifolds, orthogonal groups of signature $(2, 4)$, $(2, 5)$, $(2, 20)$, and $(2, 21)$ arise. See [5], which provides a review of the authors' results for the cases of signatures $(2, 19)$, $(2, 20)$, and $(2, 21)$. Four-dimensional moduli spaces of polarized generalized Kummer manifolds (the case of signature $(2, 4)$) are perhaps the most interesting to date.

We put

$$L_{2d, 2n+2} = U \oplus U \oplus \langle -2d \rangle \oplus \langle -(2n+2) \rangle, \quad \text{sign } L_{2d, 2n+2} = (2, 4).$$

We denote by $v_{-(2n+2)}$ the generator vector of the lattice $\langle -(2n+2) \rangle$ and by $\sigma_{-(2n+2)} \in O^+(L_{2d, 2n+2})$ the reflection defined by the vector $v_{-(2n+2)}$. We consider the double extension of the stable special orthogonal group

$$\Gamma_{2d, 2n+2} = \langle \widetilde{SO}^+(L_{2d, 2n+2}), \sigma_{-(2n+2)} \rangle.$$

Question. The lattice $L_{2d, 2n+2}$ does not satisfy the Kneser conditions from §3. Calculate the abelianization of $\Gamma_{2d, 2n+2}$.

One can show that the moduli space of generalized Kummer varieties of type $\text{Kum}^n(A)$ with a split polarisation of degree $2d$ is a four dimensional modular variety

$$\mathcal{M}(\text{Kum}^n, h_{2d}^{\text{split}}) = \Gamma_{2d, 2n+2} \backslash \mathcal{D}(L_{2d, 2n+2}),$$

where $\mathcal{D}(L_{2d,2n+2})$ is the homogeneous domain of type IV and dimension 4, associated with $L_{2d,2n+2}$. Using the theory of theta-blocks and the author lifting construction from [1] we prove

Theorem. (See [8].)

$$h^{4,0}(\mathcal{M}(Kum^2, h_{2d}^{split})) > 0, \quad \text{if } 2d \neq 2, 4, 6, 10, 12, 18;$$

$$h^{4,0}(\mathcal{M}(Kum^n, h_6^{split})) > 0, \quad \text{if } n \neq 2, 4, 5, 8.$$

In particular, in these cases the fourth cohomology group of $\Gamma_{2d,2n+2}$ is not trivial.

In [8] we give a much more general result for a very wide set of pairs of dimensions $2n$ and polarisations $2d$.

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On the Plotkin problems of Ind scheme automorphisms

Alexey Kanel-Belov

Boris Plotkin conjectured that every automorphism of endomorphisms group $s \text{ } End(k[x_1, \dots, x_n]), End(k\langle x_1, \dots, x_n \rangle)$ are (quasi) inner . i.e conjugation by automorphism may be together with mirror mapping conjugation. There are also problems on Ind-schemes automorphisms which were resolved for $Aut(k[x_1, \dots, x_n]), Aut(k\langle x_1, \dots, x_n \rangle)$ by author and A.Elishev. We discourse these conjectures and endomorphisms of semigroups $End(k[x_1, \dots, x_n]), End(k\langle x_1, \dots, x_n \rangle)$.

The talk is dedicated to the memory of Boris and Zhenya Plotkin.

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Three faces of mitosis: types A , B/C and D

Valentina Kiritchenko and Ekaterina Nistyuk

Abstract. Mitosis was first devised in type A by Allen Knutson and Ezra Miller in the context of Schubert polynomials and pipe dreams. The notion of pipe dreams can be extended to other classical types using Kogan faces of Gelfand-Zetlin polytopes. In this context, mitosis can be viewed as a convex geometric incarnation of divided difference and Demazure operators from Schubert calculus and representation theory. We will discuss mitosis and its applications in all types including the recent results of Ekaterina Nistyuk (née Presnova) in type D . All necessary definitions will be given in the talk.

Introduction

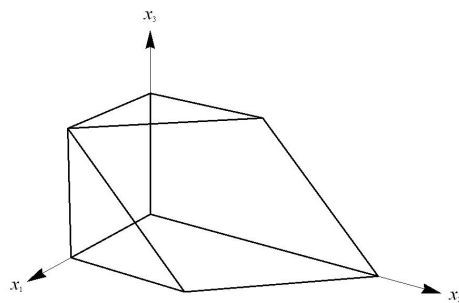
Mitosis as a mathematical notion was introduced by Knutson and Miller to generate combinatorial objects, called reduced pipe dreams or rc-graphs, from a single object. Originally, pipe dreams were used to enumerate the coefficients of Schubert polynomials in the type A . Recall that Schubert polynomials are defined inductively by applying divided difference operators to a single Schubert polynomial. Hence, mitosis operations can be thought of as combinatorial counterparts of divided difference operators from Schubert calculus.

We define a convex geometric version of mitosis. It can be applied to Gelfand-Zetlin polytopes in the classical types and other string polytopes. In types A and B/C , this construction yields already known mitosis algorithms of Knutson-Miller and Fujita, respectively. In type D , we get a new construction of mitosis on pipe dreams and reduced subwords.

1. Tables and Figures

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FIGURE 1. Gelfand–Zetlin polytope in type A_2

- [2] Ekaterina Presnova, *Gelfand–Zetlin polytopes in the type D*, Master Thesis, HSE University, 2025

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Inducing graph invariants from the universal $\mathfrak{gl}$ -weight system

Sergei Lando

Abstract. Weight systems, which are functions on chord diagrams satisfying certain 4-term relations, appear naturally in Vassiliev's theory of finite type knot invariants. In particular, a weight system can be constructed from any finite dimensional Lie algebra endowed with a nondegenerate invariant bilinear form [5]. Recently, M. Kazarian suggested to extend the $\mathfrak{gl}(N)$ -weight system from chord diagrams (treated as involutions without fixed point) to arbitrary permutations, which led to a recurrence formula allowing for an effective computation of its values, elaborated by Zhuoke Yang [6] (see also [1]). In turn, the recurrence helped to unify the $\mathfrak{gl}(N)$ weight systems, for $N = 1, 2, 3, \dots$, into a universal $\mathfrak{gl}$ -weight system. The latter takes values in the ring of polynomials $\mathbb{C}[N][C_1, C_2, \dots]$ in infinitely many variables $C_1, C_2, \dots$ (Casimir elements), whose coefficients are polynomials in N .

The universal $\mathfrak{gl}$ -weight system carries a lot of information about chord diagrams and intersection graphs. The talk will address the question which graph invariants can be extracted from it. We will discuss the interlace polynomial, the enhanced skew-characteristic polynomial, and the chromatic polynomial. In particular, we show that the interlace polynomial of the intersection graphs can be obtained by a specific substitution for the variables $N, C_1, C_2, \dots$. This allows one to extend it from chord diagrams to arbitrary permutations.

Questions concerning other graph and delta-matroid invariants and their presumable extensions will be formulated.

The talk is based on a work of the speaker, M. Kazarian and a PhD student Nadezhda Kodaneva [2, 3, 4].

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Vibemathed: the AI takeoff in mathematics

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Abstract. Between April and August 2026, general-purpose large language models (LLMs) settled a number of long-standing open problems, including the Erdős unit distance conjecture, the cycle double cover conjecture, the Jacobian conjecture, the Crouzeix conjecture, and the existence of non-sofic groups, in many cases with proofs formally verified in Lean. This extended abstract surveys these results, describes the shift from classical automated theorem proving to reasoning models and agentic workflows that made them possible, and presents a first-hand case study: an AI-assisted research program on the Dinitz–Garg–Goemans conjecture on cost-preserving single-source unsplittable flows, which produced three preprints in three weeks after the conjecture had been refuted by a model-generated counterexample. We conclude with a discussion of the community’s reactions, from the Leiden Declaration to the proposals of Tao and Gowers to redirect human effort towards “proof digestion”, and of the questions these developments raise about what a mathematical result is and whom it belongs to.

Introduction

The relationship between mathematics and automated reasoning has long been somewhat paradoxical. Since Frege, Hilbert, Gödel, and Gentzen it has been understood how to formalize essentially all of the mathematics practitioners care about, and since Robinson’s resolution principle [1] it has been known how to search for proofs mechanically. Yet six decades of automated theorem proving produced remarkably few results that a working mathematician would call interesting; the standard example remains the Robbins conjecture on Boolean algebras, proved in 1996 by McCune’s prover EQP after eight days of search [2].

The current breakthrough has arrived from the opposite direction: unlike provers that search a formal space, language models are trained on the written record of *human* mathematics and subsequently taught to *reason* in natural language. This note has three goals. First, we will survey the results of spring and summer of 2026, when publicly available LLMs began to settle problems that had been

open for decades (Sections 1–2). Second, we present a first-hand case study, the author’s AI-assisted work on the Dinitz–Garg–Goemans conjecture in network flows (Section 3). Third, we review the reactions of the mathematical community and some questions they raise (Sections 4–5). We restrict attention to models that any mathematician can use through a subscription or an API, and do not consider bespoke systems such as AlphaEvolve that require computational resources unavailable outside a few companies.

1. From proof search to reasoning models

Two developments separate the models of 2026 from GPT-4. The first is *reasoning*: starting with chain-of-thought prompting [3], and then with reinforcement learning on verifiable answers in OpenAI’s o1 and DeepSeek-R1 [4], models were trained to spend a long time thinking (writing on a scratchpad) before answering. The second is *agentic scaffolding*: tools such as Claude Code or Codex do not change the model at all, but provide harnesses that let the model work autonomously for hours, write and run code, test hypotheses, and check their own outputs. By July 2025, three systems had reached gold-medal level at the International Mathematical Olympiad [5].

The first controlled experiments with new theorems date from autumn 2025. Diez, da Maia, and Nourdin [6] asked GPT-5 to upgrade a qualitative fourth moment theorem in the Malliavin–Stein framework to a quantitative one; the model produced a correct theorem after two rounds of corrections, then proposed and proved a Poisson version; still, the authors described the model as “an executor” responding to prompts. The “Gödel test” of Feldman and Karbasi [7] posed five easy open conjectures in combinatorial optimization; GPT-5 solved three, with minor flaws and somewhat inelegant proofs. In February 2026, Knuth [8] reported that Claude Opus 4.6 had solved, in about an hour, a problem on Hamiltonian decompositions of directed Cayley graphs that he had attacked for weeks. In the spring, AI-generated solutions began to accumulate in the Erdős problems database [9], and Tao started a page listing them [10]; many early entries were literature findings rather than new proofs, and the goalposts were moved accordingly.

2. The takeoff: April–August 2026

Unit distances. Let $\nu(n)$ be the maximum number of unit distances among n points in the plane. The square lattice, together with the fact that primes $p \equiv 1 \pmod{4}$ split in the Gaussian integers, gives Erdős’s bound $\nu(n) \geq n^{1+c/\log \log n}$ [11], which he conjectured to be sharp up to the constant; the best upper bound, $O(n^{4/3})$ [12], has stood since 1984. In April 2026, GPT-5.4 Pro, prompted by a user with no advanced mathematical training, produced explicit point sets with $\nu(n) \geq n^{1+\delta}$ for a fixed $\delta > 0$; the argument was checked, corrected, and written up by Tao, Lichtman, and Alexeev, and released by OpenAI in May [13, 14], after which Sawin

made the exponent explicit, about 1.014 [15]. Gowers wrote that, had a human submitted the argument to the *Annals*, he would have recommended acceptance without hesitation [14].

Cycle double cover. The conjecture that every bridgeless graph admits a family of cycles covering each edge exactly twice was posed in the 1970s [16, 17] and remained open for snarks. On July 10, OpenAI released a complete proof produced by its model GPT-5.6 Sol Ultra [18], with an independent exposition by Oum following within a week [19]; the argument is elementary, reducing the problem via a new construction of edge colourings to a statement of linear algebra.

The Jacobian conjecture. Keller’s question of 1939 [20] asks whether a polynomial map $F: \mathbb{C}^n \rightarrow \mathbb{C}^n$ with constant nonzero Jacobian determinant must be invertible. On July 20, Levent Alpöge, a mathematician working at Anthropic, announced [21] that Claude Fable 5 had found a counterexample for $n = 3$: three explicit polynomials in x, y, z with Jacobian determinant -2 that map three distinct points to the same point (see [22] for a discussion). This made a lot of noise: verification takes minutes, but the model used was an internal one and the prompts were not released.

Astra and the flood. On August 1, OpenAI announced ten results obtained by an internal model code-named Astra [23], among them an explicit non-sofic group, improved upper bounds for sphere packings in high dimensions, a disproof of Connes’ rigidity conjecture, superpolynomial arithmetic circuit lower bounds, and Ehrhart’s volume conjecture, each accompanied by a complete Lean formalization. Results also arrived from unexpected directions. The Crouzeix conjecture in operator theory [24] was proved with ChatGPT 5.6 by Jin, a neurosurgery resident [25]; Townsend and Greenbaum [26] confirmed the argument, and Lorist and Schwenninger [27] gave a different short AI-assisted proof. On August 10, Anthropic announced a Lean-verified proof, found autonomously by Claude and written up by Alpöge and Furman, that more than two thirds of the zeros of the Riemann zeta function are simple and lie on the critical line [29, 28], improving the long-standing record of $5/12$ [31, 32]; Maynard commented that the problem needed a genuinely new idea, which this result seems to provide [30]. There is no reason to expect this progress to stop: the true difficulty of the problems that remain open is unknown.

3. A case study: the Dinitz–Garg–Goemans conjecture

The results above were obtained mostly by companies or individuals with early access to frontier models. This section describes a smaller project carried out by the author with publicly available models over three weeks in July–August 2026; full details are given in three preprints [33, 34, 35], a code repository with exact-arithmetic certificates [36], and two expository posts [37, 38].

The problem. In the *single-source unsplittable flow* problem [39], a directed graph $G = (V, A)$ has a source s and terminals $t_1, \dots, t_k$ with demands $d_1, \dots, d_k$; each arc a carries a per-unit cost c_a . A *fractional* flow x may split each demand among

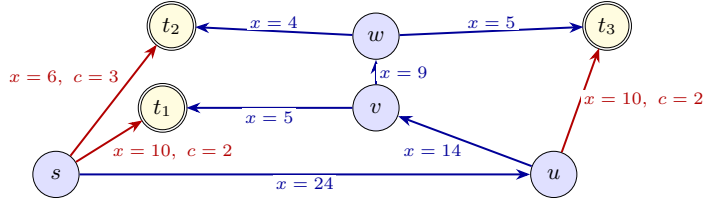


FIGURE 1. The counterexample found by Rybin and GPT-5.6 Pro: demands $d_1 = 15$, $d_2 = 10$, $d_3 = 15$; blue arcs are free, red arcs cost c per unit. The shown flow x costs 58, but every unsplittable flow with $y_a \leq x_a + D$ must use ≥ 2 paid routes and costs ≥ 60 .

several s - t_i paths; an *unsplittable* flow y must route each demand along a single path. Let $D = \max_i d_i$.

Theorem 1 (Dinitz, Garg, Goemans [40]). *For every feasible fractional flow x there exists an unsplittable flow y , computable in polynomial time, with $y_a \leq x_a + D$ for every arc a .*

Conjecture 2 (Goemans, 1999). *The flow y can be chosen with $\sum_a c_a y_a \leq \sum_a c_a x_a$.*

The conjecture was proved for demands forming a divisibility chain [41], extended in [42], established with the doubled slack $2D$ for planar graphs [43], and proved for series-parallel graphs in June 2026 [45]; a paper of October 2025 still called it “a famous conjecture” and noted that even weakened variants were open [44].

The counterexample. On July 22, 2026, Rybin posted a four-prompt transcript where GPT-5.6 Pro, after three rounds of “I have not found a complete counterexample” and a final instruction to stop reporting partial results, produced the instance of Figure 1 [46]. Its mechanism is a *triangle of conflicts*: each terminal has an expensive direct route and a free detour, any two free detours together overload a shared arc beyond $x_a + D$, while the fractional flow uses the three detours with fractions $\frac{1}{3} + \frac{2}{5} + \frac{1}{3} = \frac{16}{15} > 1$, which no unsplittable flow can imitate. Verification is simple and exhaustive since an unsplittable flow here is just a choice of one of two paths per terminal; the counterexample has been formalized in Isabelle [47].

The relaxed constant. Once the conjecture is false, the natural question is the smallest C such that a cost-preserving unsplittable flow with $y_a \leq x_a + C \cdot D$ always exists. Rybin’s instance gives $C \geq 16/15$, and the first result of the project was a proof that the triangle mechanism cannot give more than $9/8$. Everything beyond that came from a family we call *two-path instances*: a free directed spine with terminals attached by an early paid exit and a late free exit, so that an instance is encoded by a system of intervals on the spine together with demands and the fractional split. Within this family the project obtained a planar four-terminal instance with $C \geq (299 - 41\sqrt{41})/32 \approx 1.1397$ [33], a seventeen-terminal instance with $C \geq 1.28249$ [34] (Figure 2), improved by a community pull request

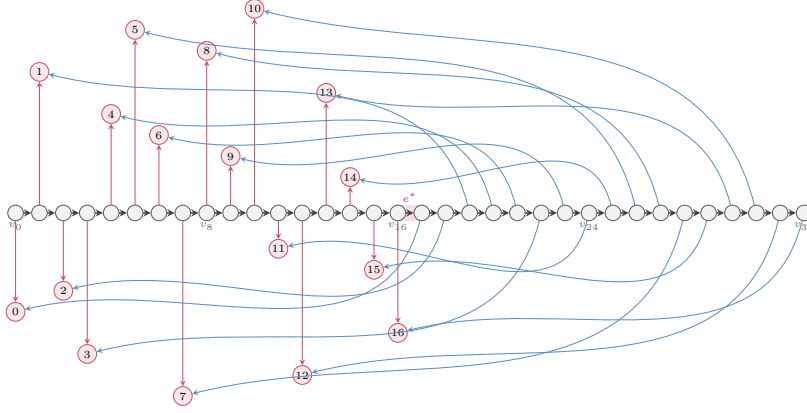


FIGURE 2. The record instance from [34]: 34 spine vertices, 17 terminals, 67 arcs. Each terminal i is drawn at its left endpoint; red arcs are early (paid) exits, blue arcs are late (free) exits; the two paths $s \rightarrow t_i$ differ by an interval of spine arcs, and all 17 intervals pass through the shaded common arc e^* .

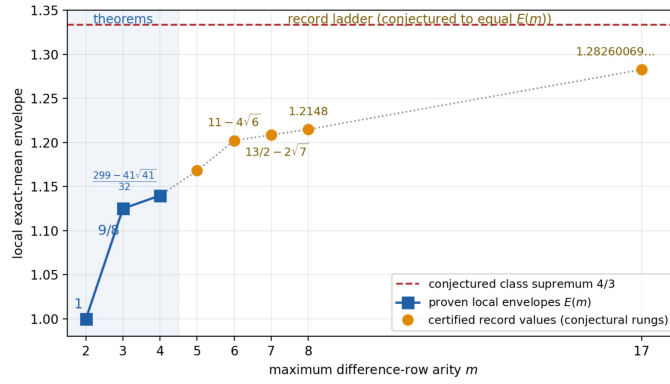


FIGURE 3. Lower bounds (record instances, circles) and proven exact local constants $E(m)$ (squares) as functions of the maximum arity m of an arc in two-path instances.

from Alfredo De la Fuente to 1.28260, and a planar six-terminal instance with $C_{\text{planar}} \geq 1.17353$, to be compared with the upper bound 2 of [43].

The third preprint [35] develops the theory behind these numbers. Its starting point is an idea already present in the literature: represent the fractional flow as a convex combination, an *exact mean*, of unsplittable flows lying in a “box”

around x . We prove that for every class of instances closed under arc deletion, the optimal cost-preserving constant times D equals the minimal radius of such a box, so that costs disappear from the problem and only geometry remains. This yields results in two directions. On the arithmetic side, a factorization principle generalizes Skutella’s divisibility-chain theorem, and a *merger reduction* shows that a finite universal constant exists if and only if the *window constant* K^* , defined over instances whose demands all lie in an interval $[n, 2n]$, is finite, with $K^* \in [C, 2C]$. On the structural side, for two-path instances the difference matrix of the two routes is a network matrix, hence totally unimodular, and the key parameter is the *arity* of an arc, the number of terminals whose switch affects it. We prove $E(2) = 1$, $E(3) = 9/8$ and $E(4) = (299 - 41\sqrt{41})/32$ for the exact local constants (Figure 3), so that the record instances of the first two preprints are optimal for their local interaction pattern, and, using Property B of hypergraphs [48] together with the classical $\lceil 3\Delta/2 \rceil$ bound for colouring paths of load Δ in a tree [49, 50], that arity at most three forces radius at most $2D$. Whether a finite universal C exists remains open; the two-path record ladder appears to converge to $4/3$, and the planar constant lies in $[1.1735, 2]$.

How the work unfolded. This was an iterative collaboration with a division of labour. GPT-5.6 Sol ran experimental “campaigns”: searching for instances, testing conjectures numerically, and producing certificates in a prescribed format. Claude Fable 5 and Claude Opus 5 handled theory, verification, and writing; they checked every campaign, proved what could be proved, and drafted the text. There were about eight such cycles of campaign, verification, theory, and a new brief. The models proposed most of the key ideas; the author chose directions, closed dead ends, and personally checked and rewrote the proofs. Errors were frequent: incorrect identities, elegant theorems refuted by the next campaign, plausible proofs with gaps... but almost all of them were caught by cross-checking one model against another, and after all models agreed the author found no further errors. The whole project fit into ordinary subscriptions costing a few hundred dollars. The outcome is not a solved conjecture but, by my personal conservative estimate, the equivalent of a solid PhD thesis produced in three weeks of part-time work.

4. Reactions

The site vibemathed.com [51] now tracks AI-solved problems by verification level, from machine-checked down to unreviewed and contested.

The *Leiden Declaration on AI and Mathematics* [52], published on June 2, 2026 and endorsed by the IMU, formulates the values of mathematical research: proof is central; results have authors who receive credit and bear responsibility; proofs are transparent and independently verifiable; results should actually be checked; and understanding within the community matters as much as the result itself. Tao’s ICM 2026 lecture [53] argues that a correct proof is not the end of mathematical cognition, and that after generation come verification, exposition,

publication, and canonicalization; he proposes to shift human effort from generating proofs and racing for priority to *proof digestion*. Gowers [54] declined to sign the Declaration, objecting to the axiom of human authorship (who proved the unit distance theorem?), but shares the concern about mathematical culture: if a pandemic wiped out all mathematicians and nobody else, the literature would remain but nobody would know what to do with it. Hairer’s post on the collective blog *Proofs and Prompts* [55] gives practical rules: be far more suspicious of AI proofs than of human ones, disclose the role of AI fully, look for primary sources, and never paste model output into a paper.

The often-made comparisons with chess are, in my opinion, misleading: chess is a competition between people, and olympiads will likewise survive the fact that several frontier models solved all six problems of IMO 2026 [56]. Research has a goal outside the competition, and a hypothetical *science* of chess would have had no place for humans for twenty years. The most radical reaction came from Tsimerman, who announced right after receiving the Fields Medal that he was moving to AI safety [57, 58], a field that badly needs mathematicians.

5. Discussion

What is a result? Tao and Gowers suggest that humans will move to digestion and explanation. This is reasonable, but it is unclear whether mathematicians will want to do this, and why models will not do it better; Tao himself asks whether we could end up with a verified proof of a major result that no human understands well enough to explain.

Whose result is it? The Declaration affirms the humanity of authorship; Gowers notes that the unit distance solution has not been credited to any human and should not be. Tao suggests that priority should go to whoever first explains a result rather than to whoever generates it. For a counterexample to the Jacobian conjecture, however, there is little to explain, and the case study above shows that the boundary between the model’s ideas and the author’s is genuinely blurred even when the author rewrites every proof.

Whom do mathematicians work for? Thurston [60] wrote that mathematicians find ways for people to understand and think about mathematics; Scholze said in 2026 that the goal of research is human understanding [59]. If so, LLMs are tools on the way to that goal. But mathematics is also a science of verifiable results that build on other results, and a theorem nobody understands can still be used.

Conclusion

I would like to conclude with a quote from Bertrand Russell’s *Autobiography*:

There was a footpath leading across the fields to New Southgate, and I used to go there alone to watch the sunset and contemplate suicide. I did not, however, commit suicide because I wished to know more of mathematics.

Let us hope that whatever happens at the frontier, mathematics will never lose this function.

P.S. The Navier–Stokes problem

I finished this text on September 7, 2026, and right the next day news came that I could not leave out: OpenAI announced a solution of the Navier–Stokes Millennium Prize problem [61], the second to fall after Perelman’s. In Fefferman’s formulation [63] the question is whether a smooth flow can develop infinite velocity in finite time, either on its own (statements A and B) or under a suitable external force (statements C and D). OpenAI’s model proved C and D, and in a companion result, less acclaimed but arguably deeper, found finite-time blow-up for the Euler equations with no force at all, from smooth compactly supported data.

After a decade of progress, from the non-uniqueness of weak solutions [65] to Euler singularities for $C^{1,\alpha}$ data [66] and computer-assisted blow-up with boundary [67], the mechanism that finished the problem grows out of a program of Córdoba and Martínez-Zoroa [68, 69, 70]: substitute a velocity field into the equation and call the residual the force; the difficulty is to make diverging terms cancel while the field blows up. Buckmaster and Alpöge, working for a year with Claude and Codex, obtained a smooth force for the Euler, Boussinesq, and porous-medium equations [71]. On September 1, OpenAI heard a rumour that a Millennium problem had been solved and launched about 10,000 agents of a model stronger than Astra on all of them; some 2.7M messages and 130B tokens later, the Navier–Stokes proof was found in 88 hours, plus 17 hours of Lean checking [62].

A controversy followed [73]: Buckmaster reported [72] an offer from OpenAI to let him write up the solution on condition that Alpöge, an Anthropic employee, be left out; Bubeck denies the condition [74], and my reading is that it was a miscommunication clarified on the same day. OpenAI also “could not rule out” that Buckmaster’s Codex sessions entered its training data [75], a lawyer’s phrase that the internet read as a confession. Neither priority is disputed: Buckmaster and Alpöge have theirs on Euler, OpenAI on the last step. Fefferman called Córdoba and Martínez-Zoroa the main heroes [76], and prize rules let the Institute include authors of prior important ideas, which will hopefully happen in this case.

On September 11, twenty-five Fields medalists, among them Deligne, Smirnov, and Tao, published a short letter, *A severe misalignment of AI in mathematics* [77]. Its premises are that mathematics is about the *road* to a result, and machine-written proofs are far from a textbook presentation. While I agree completely, I do not see how the conclusion follows: a hard-to-understand proof may or may not help the search for a textbook presentation, but it cannot make that search *harder*. There is a missing premise here, presumably that people will lose motivation to understand results once they do not actually obtain them, which I think is the actually interesting and, I believe, controversial claim.

For me, the most interesting point in the whole affair is OpenAI’s plot showing the model that found the solution to be far ahead of Astra, which had itself settled many important open problems a month earlier. Mathematical ability in LLMs is becoming (if not already) superhuman, and it looks like human level was never going to be an important threshold, let alone an asymptote. If in mathematics, then in AI research (it is math + programming + data processing); and if in AI research, then everywhere. So it goes.

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New approach to the Grothendieck – Serre conjecture

Ivan Panin

We will introduce the concept of Grothendieck-Serre presheaves and prove one very general theorem about them. And we will show how several well-known deep results and new ones follow from this theorem at once. Among the well-known is the Grothendieck-Serre conjecture.

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Asymptotics of Kronecker coefficients for large Young diagrams of fixed height

Nicolai Reshetikhin

A Kronecker coefficient represents the multiplicity of an irreducible representation associated with a given Young diagram in the tensor product of two irreducible representations. Thus, a Kronecker coefficient depends on three Young diagrams. The talk will demonstrate how to calculate the asymptotic behavior of these coefficients in the limit where the diagrams grow indefinitely at the same rate while the height of each diagram remains fixed.

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On the Baer–Suzuki width

Danila Revin

Abstract. A class $\mathfrak{X}$ of finite groups is called radical if every finite group G has the largest normal subgroup $G_{\mathfrak{X}}$ belonging to $\mathfrak{X}$. According to the definition by Gordeev, Grunevald, Kunyavskii, and Plotkin, for a radical class $\mathfrak{X}$, the Baer–Suzuki width $\text{BS}(\mathfrak{X})$ is the smallest nonnegative integer m such that

$$G_{\mathfrak{X}} = \{x \in G \mid \langle x_1, \dots, x_m \rangle \in \mathfrak{X} \text{ for all } x_1, \dots, x_m \text{ conjugate to } x\}$$

for every finite group G , or, if no such m exists, we set $\text{BS}(\mathfrak{X}) = \infty$. A special case of a radical class is a class that is complete in the sense of Wielandt, i.e., a non-empty class that is closed under taking subgroups, homomorphic images, and extensions. Recently, the speaker proved that the Baer–Suzuki width of an arbitrary complete class is finite. Now, we present an almost unimprovable estimate of the Baer–Suzuki width of a complete class and give examples of classes with infinite Baer–Suzuki width.

Introduction

According to Wielandt [1], a nonempty class $\mathfrak{X}$ of finite groups is said to be *complete* if $\mathfrak{X}$ is closed under taking

- subgroups ($G \in \mathfrak{X} \Rightarrow H \in \mathfrak{X}$ for every $H \leq G$),
- homomorphic images ($G \in \mathfrak{X} \Rightarrow \phi(G) \in \mathfrak{X}$ for every group homomorphism ϕ with domain G), and
- extensions ($N \in \mathfrak{X}$ and $G/N \in \mathfrak{X} \Rightarrow G \in \mathfrak{X}$ for every group G and $N \trianglelefteq G$).

For a complete class $\mathfrak{X}$, each finite group G has the $\mathfrak{X}$ -*radical* $G_{\mathfrak{X}}$ —the largest normal subgroup which belongs to $\mathfrak{X}$.

The Baer–Suzuki theorem states that *if $\mathfrak{X}$ is the class of p -groups where p is a prime, then*

$$G_{\mathfrak{X}} = \{x \in G \mid \langle x_1, x_2 \rangle \in \mathfrak{X} \text{ for all } x_1, x_2 \text{ conjugate to } x\}$$

in every finite group G . Gordeev, Grunewald, Kunyavskii, and Plotkin [3] and independently Flavell, Guralnick, and Guest [2] proved that *if $\mathfrak{X}$ is the class of*

solvable groups, then

$$G_{\mathfrak{X}} = \{x \in G \mid \langle x_1, x_2, x_3, x_4 \rangle \in \mathfrak{X} \text{ for all } x_1, x_2, x_3, x_4 \text{ conjugate to } x\}.$$

In [4, Definition 1.15], Gordeev, Grunewald, Kunyavskii, and Plotkin introduced the following definition.

Definition. Let $\mathfrak{X}$ be a class of finite group such that every finite group G has a largest normal subgroup $G_{\mathfrak{X}}$ belonging to $\mathfrak{X}$. The *Baer–Suzuki width* $\text{BS}(\mathfrak{X})$ is the smallest nonnegative integer m such that

$$G_{\mathfrak{X}} = \{x \in G \mid \langle x_1, \dots, x_m \rangle \in \mathfrak{X} \text{ for all } x_1, \dots, x_m \text{ conjugate to } x\}$$

for every finite group G , or, if no such m exists, we set $\text{BS}(\mathfrak{X}) = \infty$.

The Baer–Suzuki width of the class of p -groups is 2 and that of solvable groups is 4. Gordeev, Grunewald, Kunyavskii, and Plotkin [4, Problem 1.16] proposed to study the following problem.

Problem. Study other classes $\mathfrak{X}$ of finite groups for which $\text{BS}(\mathfrak{X}) < \infty$.

Recently [5], using the classification of finite simple groups, it was proved that the Baer–Suzuki width of every complete class is finite. Here we will give an estimate of the Baer–Suzuki width for complete classes and show that it is nearly unimprovable.

1. Unimprovable estimate for the Baer–Suzuki width of a complete class

It is clear that $\text{BS}(\mathfrak{X}) = 0$ if $\mathfrak{X}$ is the class of all finite groups. Hereinafter, we assume that $\mathfrak{X}$ is a complete class distinct from the class of all finite groups. By the Cauchy embedding, there exists a greatest natural number $\Upsilon = \Upsilon(\mathfrak{X})$ such that $\text{Sym}_{\Upsilon} \in \mathfrak{X}$. We call this number Υ *the symmetric boundary* of $\mathfrak{X}$.

By slightly modifying the arguments in [5], we can prove the following assertion.

Theorem. *Let $\mathfrak{X}$ be a complete class distinct from the class of all finite groups. Denote by $\Upsilon = \Upsilon(\mathfrak{X})$ the largest integer n such that $\text{Sym}_n \in \mathfrak{X}$ and set*

$$m = \max\{11, 2(\Upsilon - 1)\}.$$

Then, for every finite group G ,

$$G_{\mathfrak{X}} = \{x \in G \mid \langle x_1, \dots, x_m \rangle \in \mathfrak{X} \text{ for all } x_1, \dots, x_m \text{ conjugate to } x\}.$$

Corollary. *Let $\mathfrak{X}$ be a complete class distinct from the class of all finite groups. Then $\text{BS}(\mathfrak{X}) \leq \max\{11, 2(\Upsilon - 1)\}$.*

We will show that the estimate $\max\{11, 2(\Upsilon - 1)\}$ is unimprovable for $\Upsilon \geq 7$.

Conclusion

An unimprovable estimate is also known for $\Upsilon = 1$: it follows from Tyutyanov’s result [6] that if $\Upsilon = 1$, then $\text{BS}(\mathfrak{X}) = 2$. It is also easy to show that the symmetric boundary of a suitable complete class can take on any natural number except 3 (the latter follows from the fact that if $\text{Sym}_3 \in \mathfrak{X}$, then $\text{Sym}_4 \in \mathfrak{X}$). Thus, the unimprovable estimate for the Baer–Suzuki width in terms of the symmetric boundary Υ of a complete class is known, except for the four cases where $\Upsilon \in \{2, 4, 5, 6\}$.

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Waring-like Problems on Algebras

Anupam Singh

Solving equations is a fundamental problem in Mathematics and has wide applications in Physics and computer science. This curiosity led to the Waring problem in number theory, and more generally, over rings; the Waring-like problem over groups; Ore's conjecture; and the L'vov-Kaplansky conjecture, etc. Let $\mathcal{A}$ be an associative (need not be commutative) algebra over a field k . A polynomial $f(x_1, x_2, \dots, x_m)$ in the ring $\mathcal{A}[x_1, \dots, x_m]$, with non-commuting variables, gives rise to a map

$$w: \mathcal{A}^m \rightarrow \mathcal{A} \quad \text{defined by} \quad (a_1, \dots, a_m) \mapsto f(a_1, \dots, a_m)$$

called a polynomial map with constant (or simply a polynomial map when the coefficients are from k). The main question is to study images of such maps (for example, sum of powers, commutators, multi-linear polynomials, etc.). When $\mathcal{A}$ is non-associative, one can also ask similar questions for certain polynomials. Further, when $\mathcal{A}$ is a Lie algebra, we consider Lie polynomials.

In this talk, we present our work in the case where $\mathcal{A}$ is a matrix algebra, a quaternion algebra, an octonion algebra, or a Lie algebra. We also present some recent work on such maps studied over matrix algebra over a finite local principal ideal ring of length 2.

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Schubert varieties for Kac-Moody groups

Alexander Sivatski

Let k be a field, $n \geq 2$. Assume that n is not divisible by $\text{char } k$, and $\xi_n \in k$. Given $\alpha, \beta \in k^*$, the central simple algebra $Q \simeq (\alpha, \beta)$ with generators i, j subjected to the relations $i^n = \alpha$, $j^n = \beta$, $ij = \xi_n ji$ is called a symbol algebra. The tensor product of two symbol algebras is by definition a bisymbol algebra. The triple of algebras (D_1, D_2, Q) is called special if D_1, D_2 are bisymbol division algebras, Q is a symbol algebra, $\exp Q = n$, and $D_2 - D_1 = Q \in Br k$. The special triple (D_1, D_2, Q) is called standard if there are symbol algebras R_1, R_2, S such that $D_1 \simeq S \otimes_k R_1$, $D_2 \simeq S \otimes_k R_2$. We give a necessary condition for the special triple to be standard. Also for any symbol algebra $Q = (\alpha, \beta) \neq 0$ we show that the special triple (D_1, D_2, Q) over the rational function field $k(u, x, y)$ is nonstandard, where $D_1 = (a, x) + (b, y)$, $D_2 = (a, ux) + (b, vy)$, $a = \alpha + \beta - u$, $b = \beta^{-1}\alpha u(\alpha + \beta - u)^{-1}$, $v = \alpha + \beta$.

Time permitted, we will pose a few open questions and discuss a related quadratic form theory problem.

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The group isomorphism problem in complexity theory

Saveliy V. Skresanov

The group isomorphism problem in computational complexity theory is a decision problem which asks whether two finite groups given by their Cayley tables are isomorphic or not. Although a simple quasipolynomial algorithm for this problem working in time $n^{O(\log n)}$ for groups of order n is known since the 70s, nothing asymptotically better has been achieved since. After L. Babai's 2015 breakthrough quasipolynomial algorithm for the notorious graph isomorphism problem, the group isomorphism problem became the main obstacle towards improving the complexity for graphs.

The talk will discuss the current status of the group isomorphism problem, the primary focus being the classes of finite groups for which there is a polynomial-time isomorphism algorithm. We will also discuss how some problems which are considered to be “wild” in algebra turn out to be “easy” in the computational setting, and how these problems naturally arise in group isomorphism.

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Воспоминания о Евгении Плоткине

Елена Бунина



Я познакомилась с Женей Плоткиным лет 25 назад на конференции в МГУ, куда он прилетел всей семьей - с папой, мамой и Таней. Я делала доклад об элементарной эквивалентности групп Шевалле над полями (две группы Шевалле над полями элементарно эквивалентны тогда и только тогда, когда у них совпадают параметры построения, а соответствующие поля элементарно эквивалентны), Женя очень заинтересовался - он разбирался и в группах Шевалле, и в теории моделей, и сказал мне после доклада: "Ты наверняка сможешь сделать то же самое над локальными и полулокальными кольцами, но над произвольными это неверно, я думаю". Теперь мы точно знаем, что это верно и над произвольными кольцами, но вот то Женино замечание направило мне в очень правильное русло и увлекло этой и смежными темами на много лет. Буквально через 1-2 года мы случайно встретились в Стокгольме, где оба были с семьями и друзьями в длинных путешествиях на пересадке, и очень друг другу обрадовались. Потом у нас были периодические встречи на конференциях и иногда переписка по научным вопросам - но знали мы друг друга все же плохо.

Все изменилось в 2022 году.

Я буквально одним днем переехала в начале марта 2022 года в Израиль, была от этого в сильном стрессе, не понимала, что я хочу делать дальше. Еще работала в Московском университете - дистанционно продолжала семестр, руководила

дипломниками и аспирантами. Написала Жене, что вот я здесь. Он тут же позвал меня в гости, я приехала к нему с младшими детьми. Женя с Таней от души накормили меня (грибами!!!), было очень тепло и приятно, а потом Женя сказал: "Ну что, ты хочешь к нам в Бар Илан?" Я почти ничего не знала про Бар Илан, мало знала про другие Израильские университеты, да и вообще в тот момент не думала о поиске работы в университете. Но Женя настойчиво сказал "Лен, у нас хорошо, я тебе помогу, да и тебе точно надо в университет, я уверен". Тогда Женя повел себя со мной как добрый родитель или опекун, хотя мы совсем не были близкими друзьями, и я просто послушалась. И я об этом никогда не жалела!!! Женя очень мне помог с Бар Иланом, и вообще с научной адаптацией в новом мире с новыми правилами, я всегда буду ему за это благодарна.

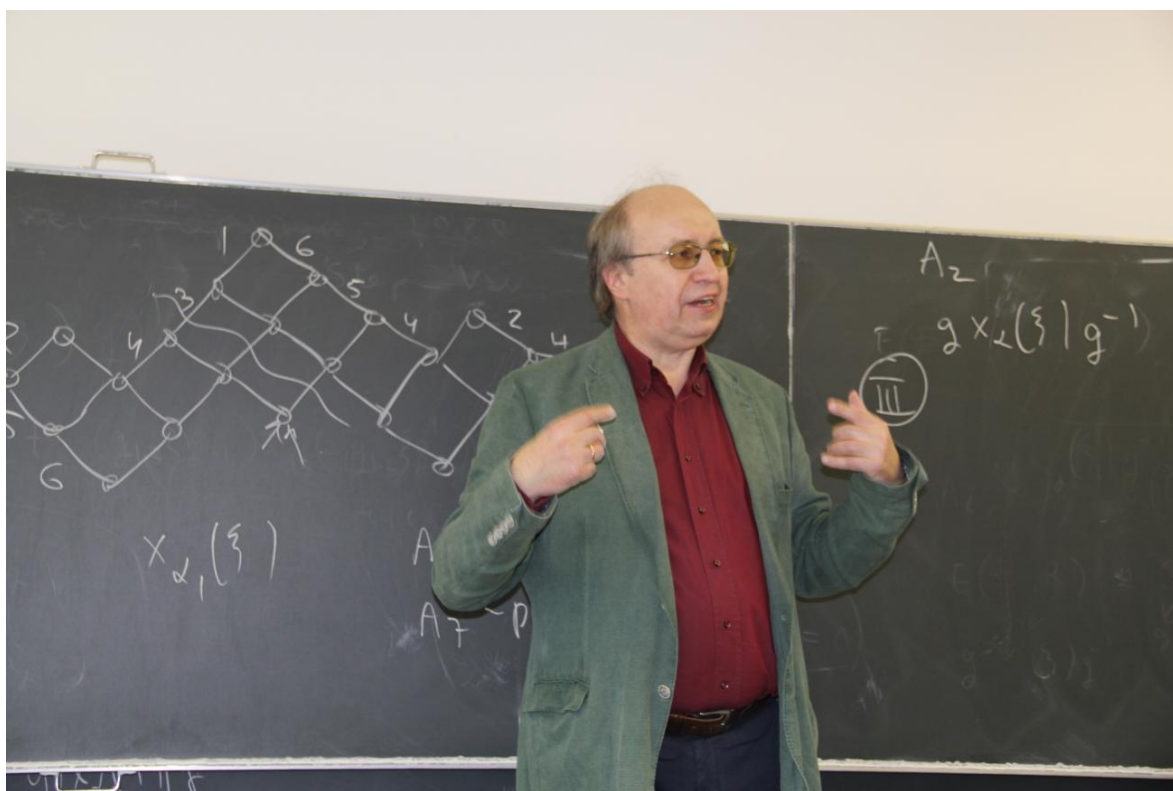
За те 4 года, что мы провели бок о бок, было много всего - мы ездили вместе на конференции, организовывали конференции вместе, писали статьи, обсуждали науку - это было очень здорово, и мне будет этого очень не хватать. Прикладываю несколько фото с наших совместных конференций - в Нью Джерси, Батуми и Тбилиси.

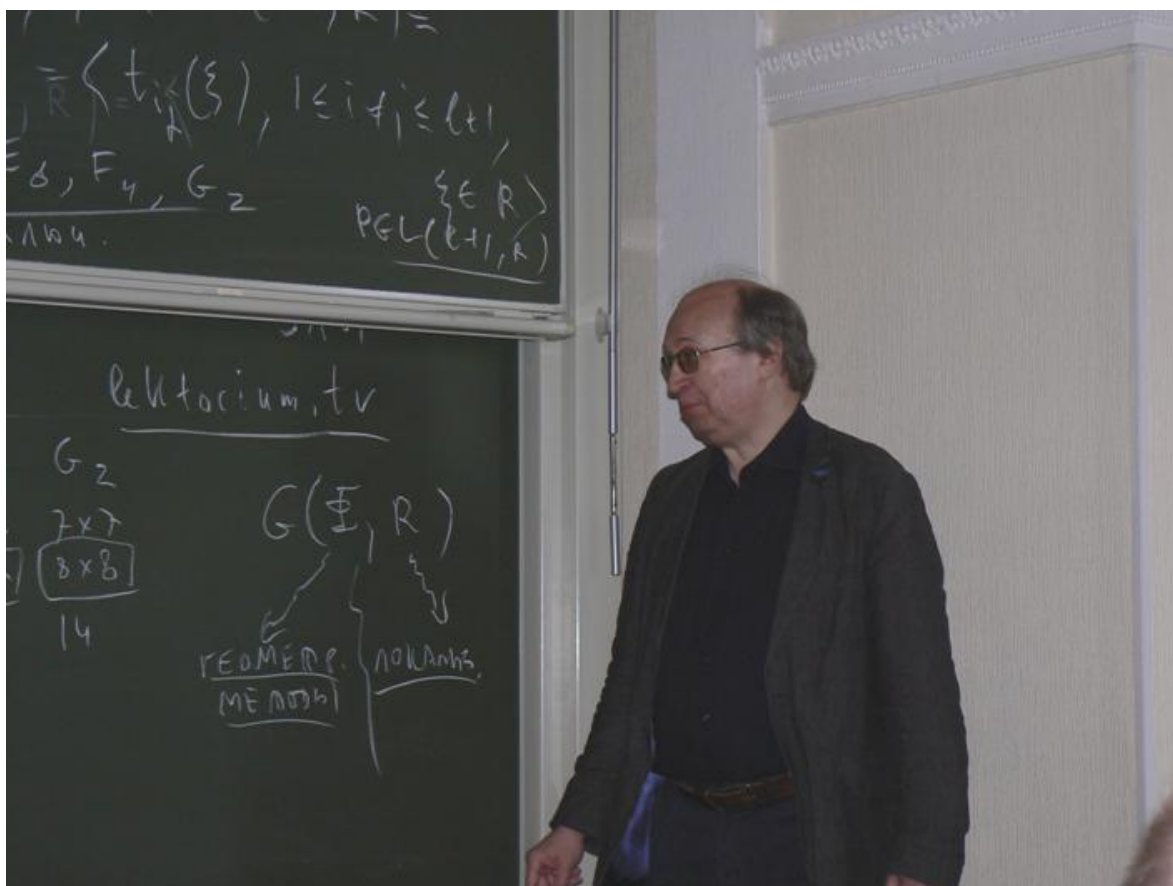
























Вспоминая друзей

Алла Григорьева



И Коля Вавилов, и Женя Плоткин всегда вызывали у нас с моим мужем Димой Григорьевым восхищение. Я училась в одной группе с Колей, он выделялся не только своими математическими способностями, но и глубокими познаниями в языках, культуре, искусстве.

Однажды в 1990 году Дима встретил совершенно неожиданно Колю на улице в Токио. Дима хотел сделать покупки, но иероглифы вместо известных ему букв привели Диму в шок, а тут вдруг встреча с Колей, который быстро разобрался с надписями, покупки были сделаны и закончились посещением хорошего ресторана!



А с Женей Плоткиным мы встречались в Международном математическом институте имени Макса Планка. Он обычно сидел допоздна, никуда не спешил, приходил на помощь всегда, сопереживал людям. Помимо математических работ, у Жени были интересные рассказы о путешествиях, которыми мы зачитывались, много архивных фотографий. Часть из них была опубликована на Международной конференции по алгебраической геометрии в августе 2026 года в его память.



Воспоминания о друге

Татьяна и Михаил Гурьевы

Женя, он такой крупный, заметный и уверенный, а внутри очень нежный, с большим сердцем и доброй душой. Если о Жене, то это - о любви. К жизни и работе, семье и друзьям. Проявляя к людям интерес, он везде обрастал знакомыми, которые множили число его друзей.

Я познакомилась с Женей в 1974 году через Таню Плоткину (тогда – Любавину). Дом ее родителей был всегда открыт для нашей группы. Мы регулярно туда приходили, советуясь по разным вопросам с мамой, Инной Яковлевной. Там же я впервые увидела длинного худого Женьку. Он приносил читать интересные книги, которые обсуждались в университетском литературном клубе. Влюбленный в Таню, он красиво ухаживал за ней. Кто бы мог устоять перед миниатюрным букетом ландышей или фиалок? Таня несла букетик на ладони, придерживая средним и безымянным пальцами.

Он сразу подружился со всеми Таниными друзьями. Желая понять его интерес к камням, мы с Галей Мурахиной, Таней и Женей ездили на свалку отходов фабрики «Русские самоцветы». Садились на Ординарной улице на троллейбус номер 1 и ехали до конца Заневского проспекта, увлеченно беседуя. А на свалке копались в отходах поделочных камней. Найдя что-нибудь интересное, бежали к Женьке, и он пояснял, что за сокровище мы нашли, и чем оно отличается от других. Иногда выходил охранник, и за несколько рублей он мог вынести нам ведро свежих каменных обрезков. Там можно было найти кусочки малахита, лазурита, оникса, нефрита. Помню, что я как-то нашла пепельницу из малахита, и была счастлива.

Когда в 90-ые годы Плоткины переехали в Ригу, мы приезжали к ним на туристическом автобусе. Шел дождь. Нас было четверо девчонок, и нам были очень рады, просушили одежду, накормили, долго с нами беседовали, и оставили ночевать.

Женя любил делиться с друзьями своими знаниями, увлечениями и чувствами. Воспринимая мир открытой душой, он хотел, чтобы и друзья чувствовали то же, что и он сам. Он любил спорить, но умел и внимательно слушать, и соглашаться с чужим мнением.

В 90-ые, когда они жили в Израиле, а мы еще не могли путешествовать, он присылал подробнейшие описания городов, которые посетил, и происшествий, в которые обязательно попадал, очень интересно описывая досто⁶³примечательности, свои приключения и впечатления. А когда стали доступны цветные фотографии,

Плоткины привезли из Турции пачку из полутысячи снимков, чтобы рассказать, что видели там. Опытный путешественник, он собирал большие команды, организуя увлекательные и тщательно продуманные поездки.

Дом Плоткиных в Израиле был всегда полон гостей. Иногда приходилось использовать даже балкон. Причем некоторые друзья жили подолгу. Непонятно, когда хозяева успевали готовиться к работе.



Женька составлял в Израиле для нас программы поездок и списки достопримечательностей. Когда мог, ездил с нами, показывал страну, стремясь передать нам свои чувства, мысли и ощущения. Он останавливался в пустыне ночью, чтобы показать горящие лунным светом глаза горных козлов; днём - чтобы рассмотреть и пощупать цветной песок, чтобы понюхать цветы и обсудить, что с ними связано, и чтобы нарвать спелых лимонов. Привозил к Красному морю, в место, где нельзя купаться и кормить рыб (там висели соответствующие таблички на РУССКОМ языке). Чем мы и занимались. А потом спрашивал: «Они кусали тебя?» И если кусали – радовался. Значит все полностью прочувствовано.



А когда он вел нам экскурсию по Иерусалиму, на звук его голоса моментально стекались слушатели. И он никогда их не отгонял.





И еще Женя очень любил петь. Вокруг него всегда были и стихи, и музыка. Он знал слова почти всех песен на свете, так же, как и Миша. Они могли петь до утра, перебирая самые забытые песни. Слух у обоих был не идеальным, но с годами практики он заметно улучшился.

Громогласный, уверенный в себе, и одновременно внимательный и деликатный, он был душой нашей веселой, шумной и пёстрой компании. Он любил нас, и это было взаимно. При встрече сразу обнимет, уведет в сторонку, расспросит, посочувствует или посмеется. Равнодушным никогда не был, а в трудные минуты просто внедрялся и порой управлял ситуацией. Как он держал в памяти все обо всех, и как ему хватало энергии – просто загадка!

Женя прожил необыкновенную, яркую интересную жизнь. Он ушел очень рано, и совершенно для нас неожиданно. Но в памяти нашей он прежний, шумный, живой и веселый. И очень добрый.