

Waring-like Problems on Algebras

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Solving equations is a fundamental problem in Mathematics and has wide applications in Physics and computer science. This curiosity led to the Waring problem in number theory, and more generally, over rings; the Waring-like problem over groups; Ore's conjecture; and the L'vov-Kaplansky conjecture, etc. Let $\mathcal{A}$ be an associative (need not be commutative) algebra over a field k . A polynomial $f(x_1, x_2, \dots, x_m)$ in the ring $\mathcal{A}[x_1, \dots, x_m]$, with non-commuting variables, gives rise to a map

$$w: \mathcal{A}^m \rightarrow \mathcal{A} \quad \text{defined by} \quad (a_1, \dots, a_m) \mapsto f(a_1, \dots, a_m)$$

called a polynomial map with constant (or simply a polynomial map when the coefficients are from k). The main question is to study images of such maps (for example, sum of powers, commutators, multi-linear polynomials, etc.). When $\mathcal{A}$ is non-associative, one can also ask similar questions for certain polynomials. Further, when $\mathcal{A}$ is a Lie algebra, we consider Lie polynomials.

In this talk, we present our work in the case where $\mathcal{A}$ is a matrix algebra, a quaternion algebra, an octonion algebra, or a Lie algebra. We also present some recent work on such maps studied over matrix algebra over a finite local principal ideal ring of length 2.

References

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